



Travelling Salesperson: Courier Service Answer Sheet

Problem

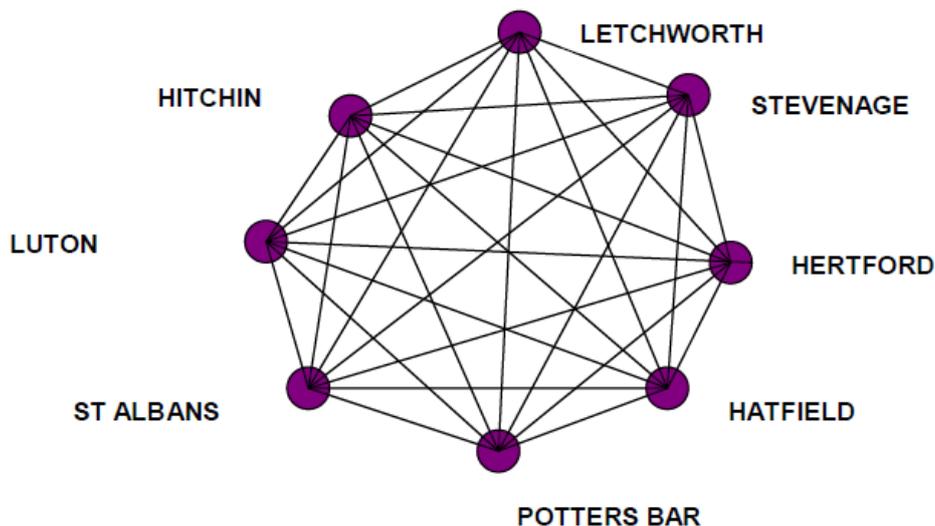
A courier service based in Luton needs to make deliveries at Hitchin, Hatfield, Letchworth, Stevenage, Hertford, St Albans and Potters Bar before returning to the central depot at Luton.

Question 1

Using the map and distance table above, work out a sensible route for the courier to take.

The table below shows the distances of the shortest routes between all the towns.

	Luton	Hitchin	St. Albans	Letchworth	Stevenage	Hatfield	Hertford	Potters Bar
Luton	-	8	12	11	14.5	14	23	22.5
Hitchin	8	-	20	3	6.5	20	11.5	29.5
St. Albans	12	20	-	23	20	6.5	15.5	10.5
Letchworth	11	3	23	-	6.5	20	17.5	29.5
Stevenage	14.5	6.5	20	6.5	-	13.5	11	21.5
Hatfield	14	20	6.5	20	13.5	-	9	9.5
Hertford	23	11.5	15.5	17.5	11	9	-	10.5
Potters Bar	22.5	29.5	10.5	29.5	21.5	9.5	10.5	-





To find a Lower Bound:

Node deleted	Total weight of MST	Total weight of two shortest edges connected to deleted node	Lower Bound
Luton	$3 + 6.5 + 6.5 + 9 + 9.5 + 11 = 45.5$	$8 + 11 = 19$	64.5
Hitchin	$6.5 + 6.5 + 9 + 9.5 + 11 + 11 = 53.5$	$3 + 6.5 = 9.5$	63
St. Albans	$3 + 6.5 + 8 + 9 + 9.5 + 11 = 47$	$6.5 + 10.5 = 17$	64
Letchworth	$6.5 + 6.5 + 8 + 9 + 9.5 + 11 = 50.5$	$3 + 6.5 = 9.5$	60
Stevenage	$3 + 6.5 + 8 + 9 + 9.5 + 11.5 = 47.5$	$6.5 + 6.5 = 13$	60.5
Hatfield	$3 + 6.5 + 8 + 10.5 + 10.5 + 11 = 49.5$	$6.5 + 9 = 15.5$	65
Hertford	$3 + 6.5 + 6.5 + 8 + 9.5 + 12 = 45.5$	$9 + 10.5 = 19.5$	65
Potters Bar	$3 + 6.5 + 6.5 + 8 + 9 + 11 = 44$	$9.5 + 10.5 = 21$	65

- Highest lower bound is 65

To find Upper Bound:

Starting vertex	Tour using Nearest Neighbour Algorithm	Weight of tour
Luton	Lu, Hi, Le, Ste, He, Ha, SA, PB, Lu	$8 + 3 + 6.5 + 11 + 9 + 6.5 + 10.5 + 22.5 = 77$
Hitchin	Hi, Le, Ste, He, Ha, SA, PB, Lu, Hi	$3 + 6.5 + 11 + 9 + 6.5 + 10.5 + 22.5 + 8 = 77$
St. Albans	SA, Ha, He, Ste, Hi, Le, Lu, PB, SA or SA, Ha, He, Ste, Le, Hi, Lu, PB, SA	$6.5 + 9 + 11 + 6.5 + 3 + 11 + 22.5 + 10.5 = 80$ $6.5 + 9 + 11 + 6.5 + 3 + 8 + 22.5 + 10.5 = 77$
Letchworth	Le, Hi, Ste, He, Ha, SA, PB, Lu, Le	$3 + 6.5 + 11 + 9 + 6.5 + 10.5 + 22.5 + 11 = 80$
Stevenage	Ste, Hi, Le, Lu, SA, Ha, He, PB, Ste or Ste, Le, Hi, Lu, SA, Ha, He, PB, Ste	$6.5 + 3 + 11 + 12 + 6.5 + 9 + 10.5 + 21.5 = 80$ $6.5 + 3 + 8 + 12 + 6.5 + 9 + 10.5 + 21.5 = 77$
Hatfield	Ha, SA, PB, He, Ste, Hi, Le, Lu, Ha or Ha, SA, PB, He, Ste, Le, Hi, Lu, Ha	$6.5 + 10.5 + 10.5 + 11 + 6.5 + 3 + 11 + 14 = 73$ $6.5 + 10.5 + 10.5 + 11 + 6.5 + 3 + 8 + 14 = 70$
Hertford	He, Ha, SA, PB, Ste, Hi, Le, Lu, He or He, Ha, SA, PB, Ste, Le, Hi, Lu, He	$9 + 6.5 + 10.5 + 21.5 + 6.5 + 3 + 11 + 23 = 91$ $9 + 6.5 + 10.5 + 21.5 + 6.5 + 3 + 8 + 23 = 88$
Potters Bar	PB, Ha, SA, Lu, Hi, Le, Ste, He, PB	$9.5 + 6.5 + 12 + 8 + 3 + 6.5 + 11 + 10.5 = 67$

- Lowest upper bound is 67
- We now know that $65 \leq \text{length of optimal route} \leq 67$
- We are unlikely to be able to improve on our route of length 67, which goes from Luton to Hitchin to Letchworth to Stevenage to Hertford to Potters Bar to Hatfield to St Albans, and then back to Luton



Question 2

What benefits are there to be gained by the courier having a 'good' solution to the problem above?

- Possible benefits include saving money, saving time and impact on the environment.

Question 3

What other factors might the courier want to take into account? Suggest additional information that it would be useful to have in order to plan the best route more realistically.

- In general, one might expect time rather than mileage to be the major factor; it would be more helpful to know the average time it takes to travel between the towns, rather than the distance. More detailed knowledge of local traffic conditions, direct routes being quicker than ones through towns etc. might also be helpful.